Investigation: Transit Tracks

Students will be able to

- describe a transit and the conditions when a transit may be seen
- describe how a planet's size and distance from its star affects the behavior of transits
- interpret graphs of brightness vs time to deduce information about planet-star systems.

Materials and Preparation

- Clip-on lamp with frosted spherically shaped low wattage (25 W maximum) bulb.
- 4 beads, various sizes (3-12mm) and colors on threads of various lengths (20-100cm). For best results, use black thread.
- "Transit Light Curves" - one set per group of 2-5 students
- Worksheets: 1 per student or 1 per group
- Account of Jeremiah Horrocks observations of the transit of Venus from http://kepler.nasa.gov/ed/lc
- Optional: light sensor and computer with sensor interface and graphing function.
- Optional: Johannes Kepler information: http://kepler.nasa.gov/Mission/JohannesKepler/

A. What is a transit?

1. Introduce students to the concept of transits by reading a short account of the first transit observation by Jeremiah Horrocks (http://kepler.nasa.gov/ed/lc/horrocks.html)

2. Demonstrate a transit by positioning the clip-on lamp at a height between standing eye-level and seated eye-level. Swing the largest bead on a thread in a circle around the lamp, with the lamp at the center in the plane of the orbit. Tell the class that the light bulb represents a star and the bead a planet; the planet is orbiting its star, like the Earth or Venus orbit the Sun.

   a. With students seated, ask if anyone can see the bead go directly in front of the star. If the lamp is high enough, no student will be able to see the bead go directly in front of the star.

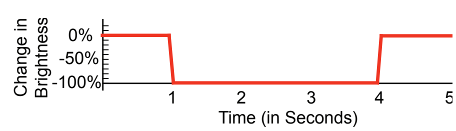
   b. Ask students to move to where they can see the bead go directly in front of the star--it's OK to stand or crouch. After a show of hands indicates everyone can see that event, confirm that is what we mean by a transit—an event where one body crosses in front of another, like when a planet goes in front of a star.

B. How does a planet's size and orbit affect the transit?

To see how planet's diameter and orbit affect transits, orbit the other beads around the light. Make those with shorter strings go in smaller radius orbits with shorter period. Define "period" as the time for one orbit. Ask the students what's different about the planets. They should identify: size, color, period, distance from the star. Ask them if there is any relationship between the planet's period and how far it is from the light. They should notice that the farther it is from the light, the longer the period.

C. Interpreting Transit Graphs

1. Imagine a light sensor. Have students imagine they have a light sensor to measure the brightness of the star (light bulb). Move a large opaque object (e.g. a book or cardboard) in front of the star so that its light is completely blocked for all the students. Ask, "If we plotted a graph of brightness vs time—with brightness measured by our light sensor—and this [book] transited the light for 3 seconds, what would the graph look like?" Have volunteers come up and draw their ideas on the board and discuss with the class. We would expect the graph to look like the one shown in Fig. 1: a drop in brightness to 100% blocked.



2. Graph for an orbiting planet. Ask the students, "What would a graph of sensor data look like for the orbiting planet, if we plotted brightness vs time?" Have volunteers draw their ideas on the board, and discuss with class. If their comments do not encompass the idea that the dips in brightness would be very narrow and that their depth would depend on the size of the beads/planets, ask them questions about how wide and deep the dips should be. We would expect the graph to look like the one shown in Fig. 2: horizontal line with dips in brightness to X% blocked.

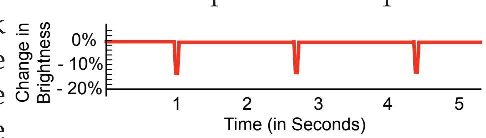


Figure 2. Light curve for a bead orbiting a light.

3. What the graphs reveal: Explain that with transit data, it's possible to calculate a planet's diameter and distance from its star. Ask, "Why do you think those two properties, planet diameter and distance from star, might be important?"
4. Analyze light curves: Hand out a set of graphs, Transit Light Curves" (pages 3-7) and the "Analyzing Light Curves" worksheet (page 8, or 11) to each group of 2-5 students, and have them interpret the graphs.
5. Discussion questions:

   How big is the planet compared with the star?

   Assuming the star is Sun-like, and that Earth would make at 0.01% drop in brightness of the Sun if it transited, how big is the planet compared with Earth?

   What is/are the period(s) of the planet(s)?

   How far is the planet from its star?

D. Extension Activity: Making Light Curves

1. Students create their own light curves, choosing planet size and orbital radius, and figuring out how to make the period for their graphs.
2. Trade light curves: Students trade light curves and challenge each other to figure out what kind of planetary system they created.

A Note on Shape of Transit Light Curves

A planet takes several hours to transit a star. The "Transit Tracks" light curves are all scaled in "days." The brightness dips appear as "spikes" rather than the "u-shaped" curve shown in figure 3 because an event that takes just a few hours is compressed when scaled against "days" of duration in the light curves. See PPT slide annotations for more inforamtion.

Optional: Collect Real Data

If you have a light sensor, computer with sensor interface, graphing software, and a computer display projector, place the light sensor in the plane of the planet/ bead orbit and aim sensor directly at the light. Collect brightness data and project the computer plot in real time. Let the students comment on what they are observing. Instead of swinging beads, you may use a mechanism, known as an orrery, to model the planets orbiting their star. Instructions for building an orrery from LEGO™ parts may be found on the NASA Kepler Mission website at http://kepler.nasa.gov/education/ModelsandSimulations/LegoOrrery/

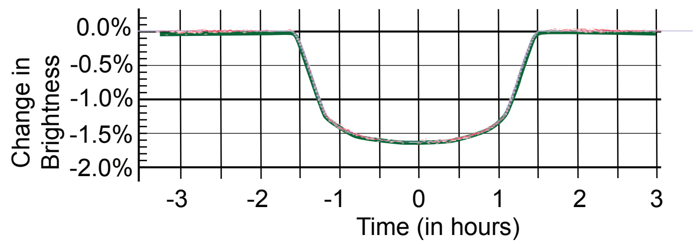


Fig. 3. Transit light curve in hours, not days

Guide to pages 4-13.

"Transit Light Curves" on pages 4 - 8, except for the "mystery" light curve (p. 6), are all are based upon real data from the Kepler Mission planets. HAT-P (Hungarian Automated Telescope network) and TrES (Trans-Atlantic Exoplanet Survey) are both ground-based telescopes surveys that discovered transiting planets before Kepler launched. Kepler looked at these three already discovered exoplanets. The "Mystery Planet" includes an Earth-size planet on a one-year orbit, and is simulated data. Duplicate one set per group; graphs can be re-used.

"Analyzing Light Curves" worksheet on page 9 is for students to record data they derive the light curves. Duplicate one per student/group, or have students set their own tables.

"Kepler's 3rd Law Graphs" Page 10 has three graphs of "Kepler's 3rd Law" scaled for planets with periods of less than 2 years. For comparison, the inner planets of our solar system are marked on the graphs. On page 12, Kepler's 3rd Law is plotted for our solar system including asteroids and the dwarf planet Pluto. All objects move on Keplarian orbits. (Note that the inner planets are close together on this linear scale, p. 12) Students analyze the light curves to determine the period (year-length) for each transiting planet. Using the "Kepler's 3rd Law Graphs," they determine the orbital distance. (Note: The distance in Kepler's 3rd Law is technically the semi-major axis of the orbital ellipse. The semi-major axis is 1/2 the major axis, which is the longest distance across the ellipse. Kepler's 3rd Law states that the period (T, for Time) is proportional to the planet's distance (R) from the Sun. R is the semi-major axis. See page 3 for further information.) Duplicate one set per group; Kepler's 3rd Law Graphs can be reused.

"Mathematics for Transit Tracks" on page 3 is for the you, the teacher—it explains the graphical and mathematical methods for analyzing the light curves.

"Analyzing Light Curves: Calculated with Kepler's 3rd Law" on page 11 is the alternative student worksheet for light curve analysis using calculations instead of graphical interpretation. Duplicate one per group or student.

Kepler's 3rd Law: Pages 12 and 13 display "Kepler's 3rd Law" as a linear and log-log graph respectively. These are for reference and can be shown to students to explain why there are three scaled graphs on page 9.

Mathematics of Transit Tracks

This is an explanation of the mathematics of Transit Tracks for the teacher. For more advanced students, you may wish to introduce mathematics in the "Tracking Transits" investigation, and have your students compute (rather than do graphical interpretation) the planet's size and distance from its star. Both methods are discussed.

PLANET SIZE

We assume that the "Transit Tracks" stars are the same size as the Sun. The graphs are normalized to solar-size stars. The radius of the planet (r_p) is calculated from the percentage drop in brightness (Z). The radius of the planet (r_p) as compared with the radius of Earth (r_e) is

$$r_p = 10\ r_e \times \sqrt{Z}$$

Note, $r_e = 1$.

Do an example and the give some examples to students as exercises: Z = 25%, 49%, 9%, 16%, 4%

For Z= 25%, r_p = 50 Earth radii (a huge planet....)

Note if Z = 49%, it's not a planet at all, but a companion star in a binary star system.

For students who finish early, challenge them to derive the formula using algebra. Have them start with the basic idea that drop in brightness is the ratio of the area of the planet (A_p) to the area of the star (A_s):

$$A_p / A_s \times 100 = Z\%$$

They can use the formula for area

$$A = \pi r^2$$

Here are the steps:

$$100 \times \pi r_p^2 / \pi r_s^2 = Z\%$$

or $\quad r_p = r_s \times \sqrt{(Z/100)} = \quad r_s \times \sqrt{(Z)}/10)$

where r_s is the radius of the star.

If the star is about the size of the Sun, then the radius of the star is about 100 times the radius of Earth (r_E) and

$$r_p = 10 r_E \times \sqrt{Z}$$

PLANET'S DISTANCE FROM ITS STAR

The distance of the planet from the star is the radius (R) of its orbit, if the orbit is a circle with the star at the center. In reality, planet's orbits are ellipses. For simplicity, we can use the special case ellipse: a circle. Johannes Kepler found that a planet's orbital radius is related to its period (T), the time it takes to orbit once. The farther out the planet is, the longer it takes to orbit. The relationship is known as Kepler's 3rd Law. Students can do the Kepler's 3rd law computation in one of two ways:

Method A: Graphical.

Use the "Kepler's 3rd Law Graphs" on page 10 to find the distance from the period determined from the light curves. These linear graphs are at different scales because if they are all graphed on the same scale (see page 12), the planet's distance is hard to interpret for short period orbits. Alternatively, students can use the log-log graph (page 13) to determine orbital distance. Many students find log-log graphs confusing. We recommend using the linear graphs (page 10).

Method B-Computational: Use Kepler' 3rd Law

For this exercise, we assume that the stars are Sun-like. Like Kepler did, we express the planet's distance in AU (Astronomical Unit = average distance from Earth to the Sun). Thus, Kepler's 3rd Law is simply:

$$R^3 = T^2 \quad *$$

or

$$R = \sqrt[3]{T^2}$$

For practice, students can calculate the orbital period from the planet's distance with these results:

T =	1 yr,	2.83 yr,	5.196 yr,	0.3535 yr
R =	1 AU,	2 AU,	3 AU,	1/2 AU

To analyze the Transit Tracks light curves using calculations, use the worksheet "Analyzing Light Curves: Calculated with Kepler's 3rd Law." Note: Students will have to convert "days" to "years" to derive the orbital distance. Both graphical and computational method that do not take into account the star's mass will give only approximate orbital distance. Use the following equation for exact results:

$$R = \sqrt[3]{T^2 \bullet M_s}$$

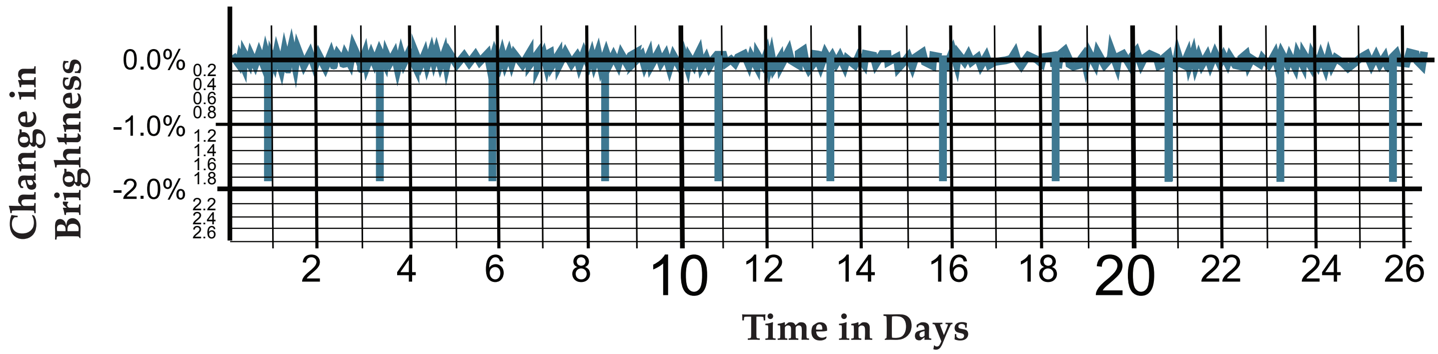
R = Orbital Distance in AU, T = Period in years
M_s = Mass of the star (Sun = 1)

Star Masses for Kepler 1 through 11:
Units = M_s where M_s = 1 for the Sun

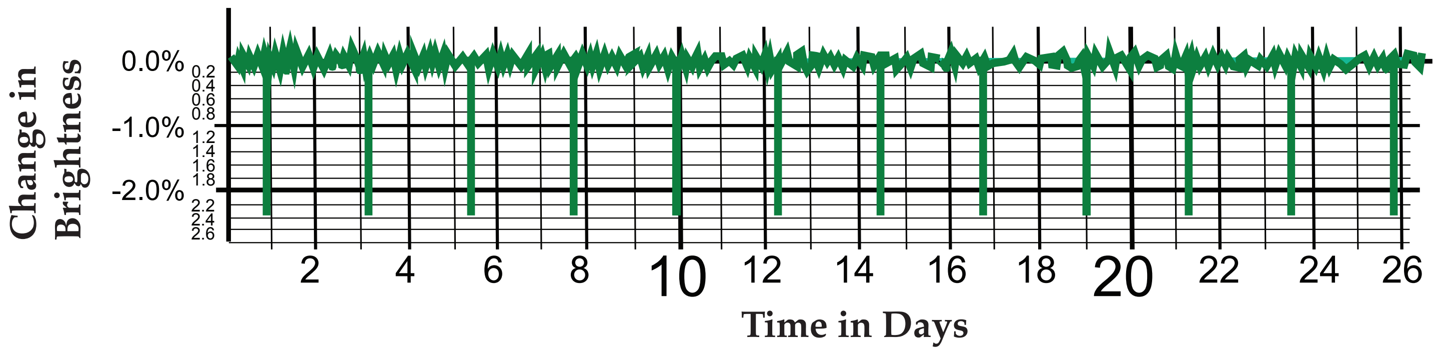
Kepler-1 = 0.98 M_s	Kepler-5 = 1.374 M_s	Kepler-9 = 1.07 M_s
Kepler-2 = 1.52 M_s	Kepler-6 = 1.209 M_s	Kepler-10 = 0.895 M_s
Kepler-3 = 0.81 M_s	Kepler-7 = 1.347 M_s	Kepler-11 = 0.95 M_s
Kepler-4 = 1.223 M_s	Kepler-8 = 1.213 M_s	

Transit Light Curves

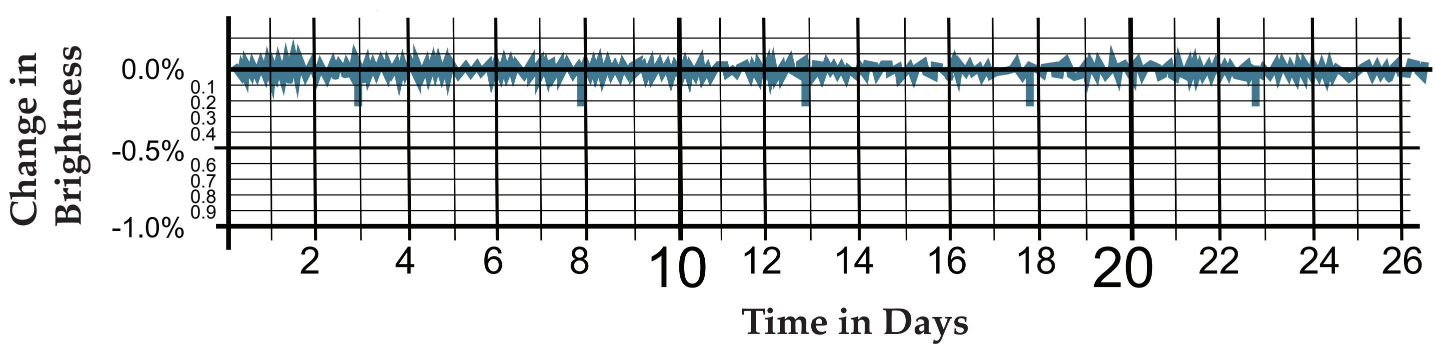
Kepler-1b (TrES-2)



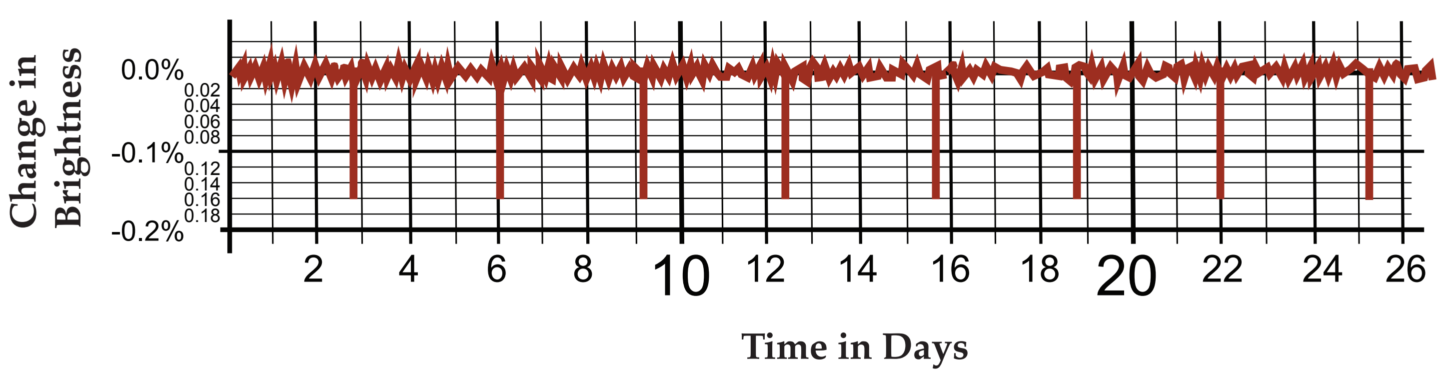
Kepler-2b (HAT-P 7b)



Kepler-3b (HAT-P-11b)

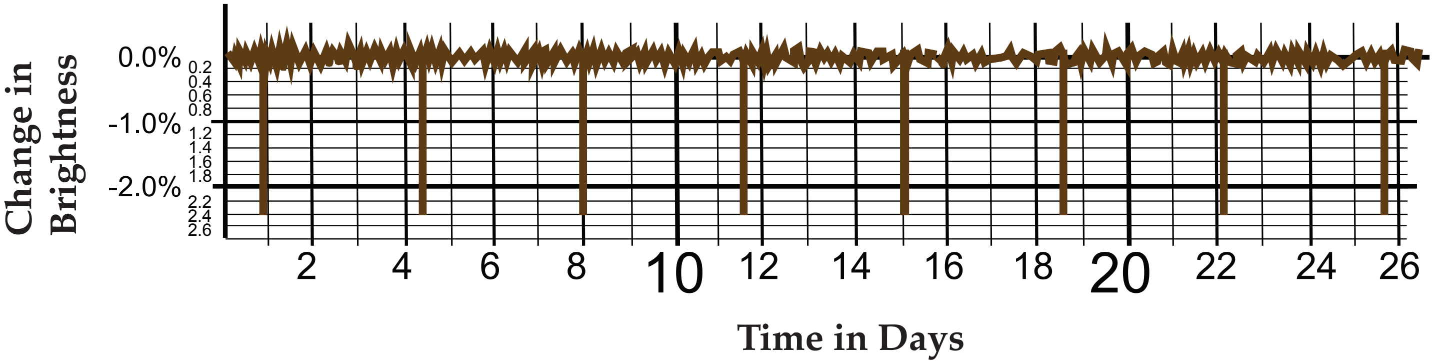


Kepler-4b

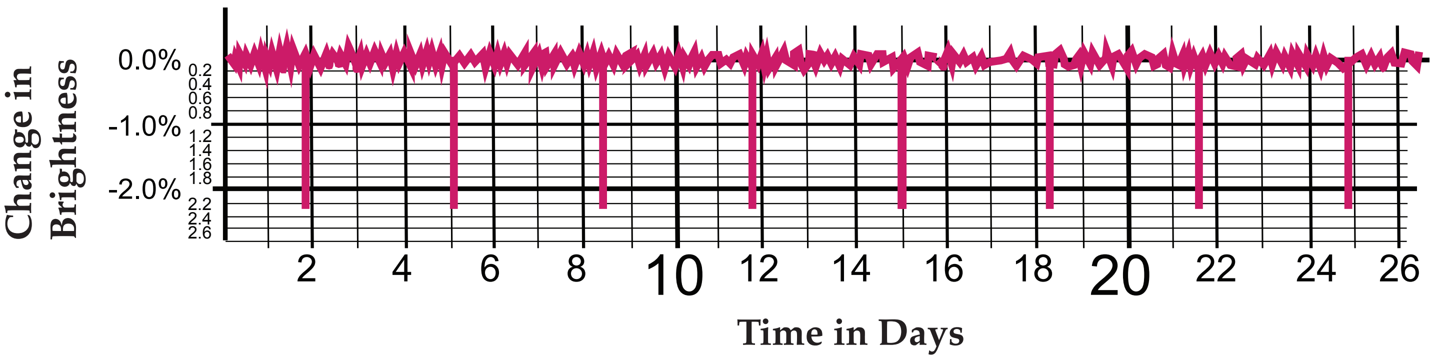


Transit Light Curves

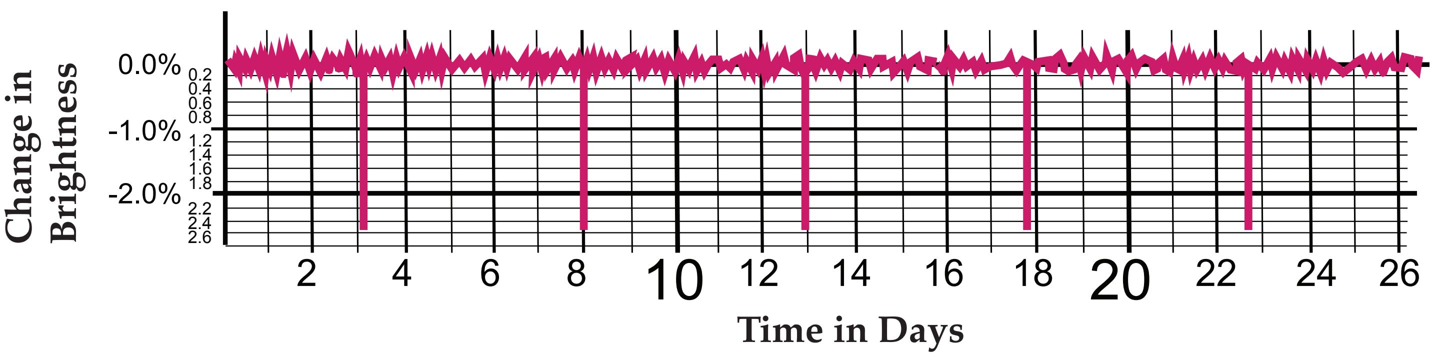
Kepler-5b



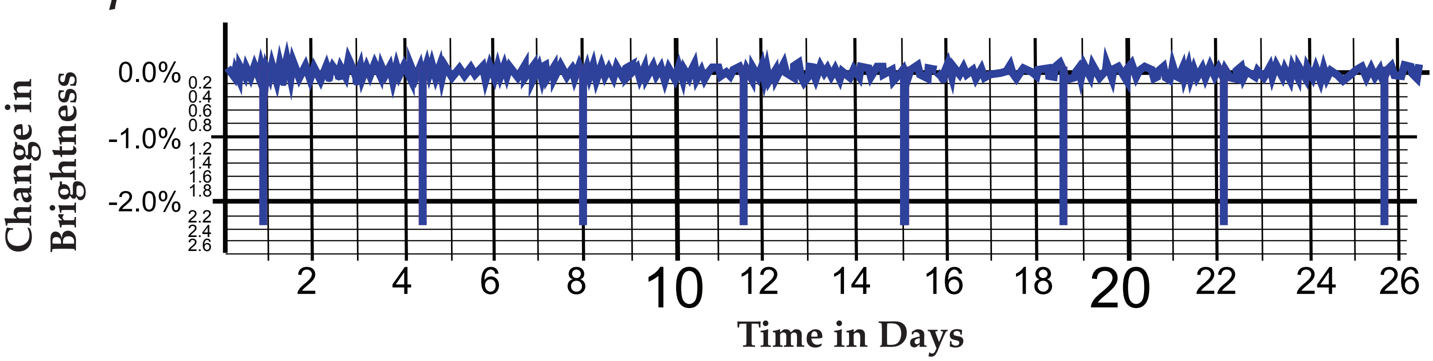
Kepler-6b



Kepler-7b

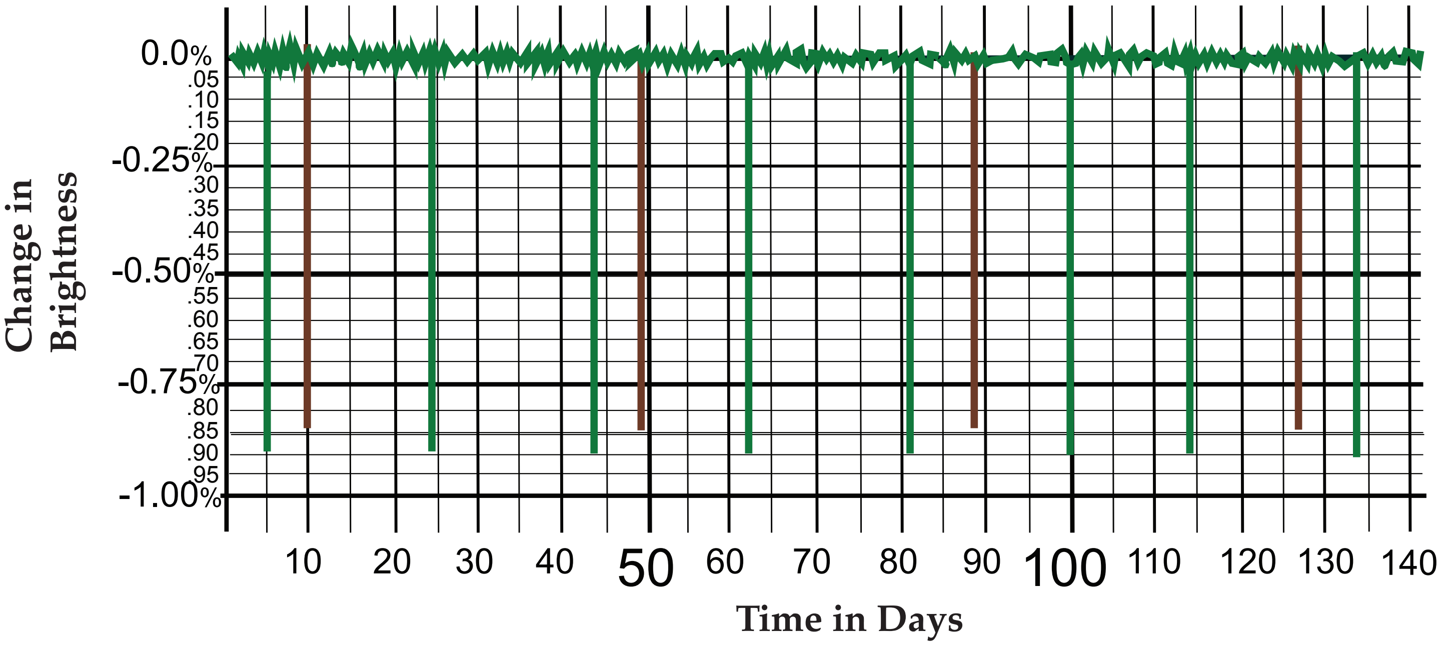


Kepler-8b

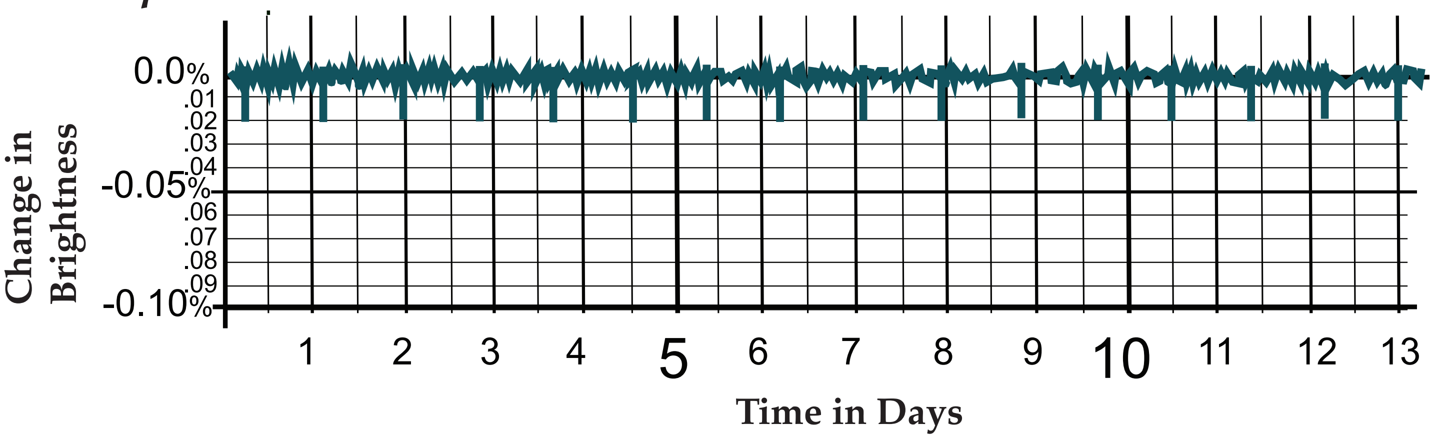


Transit Light Curves

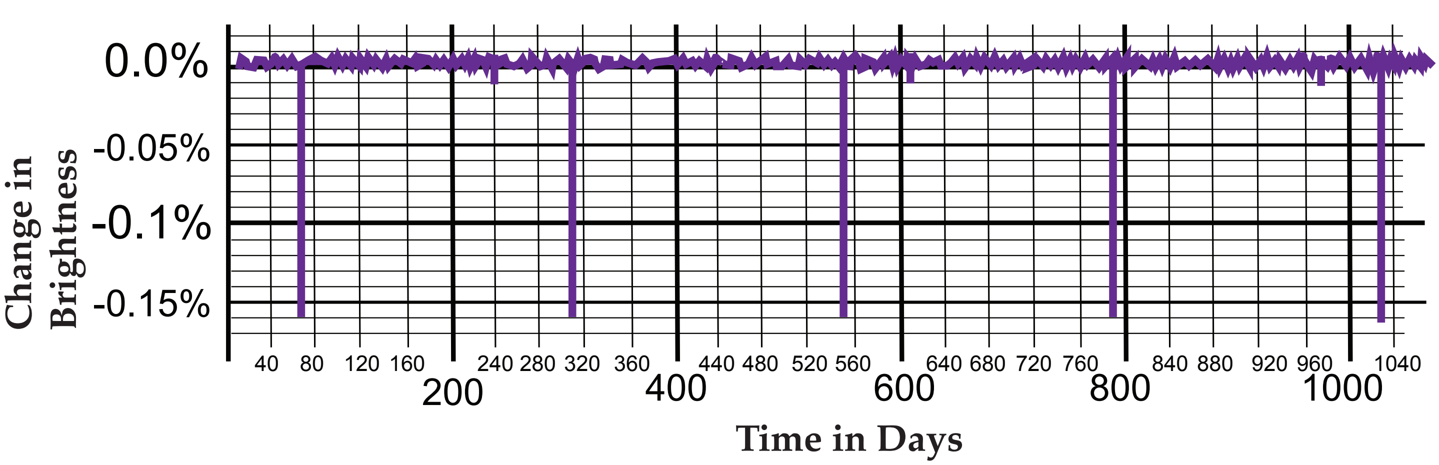
Kepler-9b, 9c



Kepler-10b

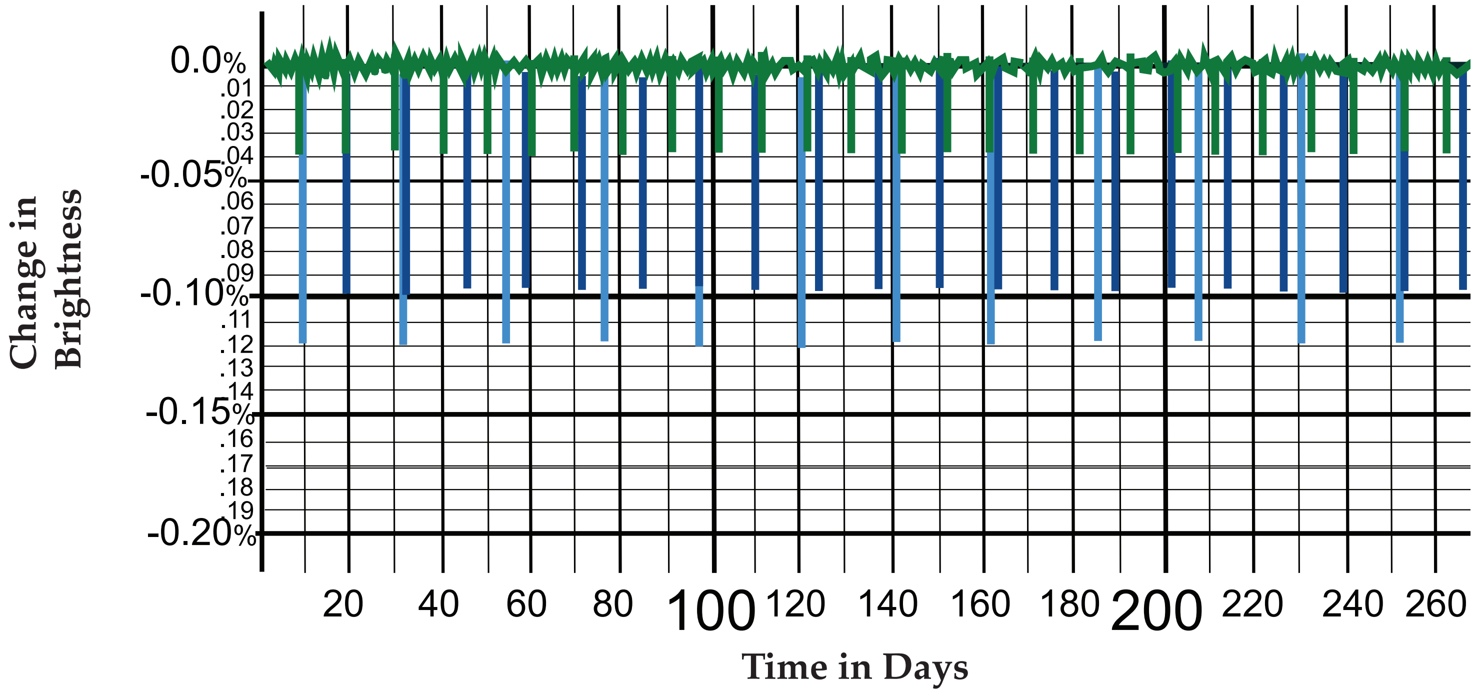


Mystery

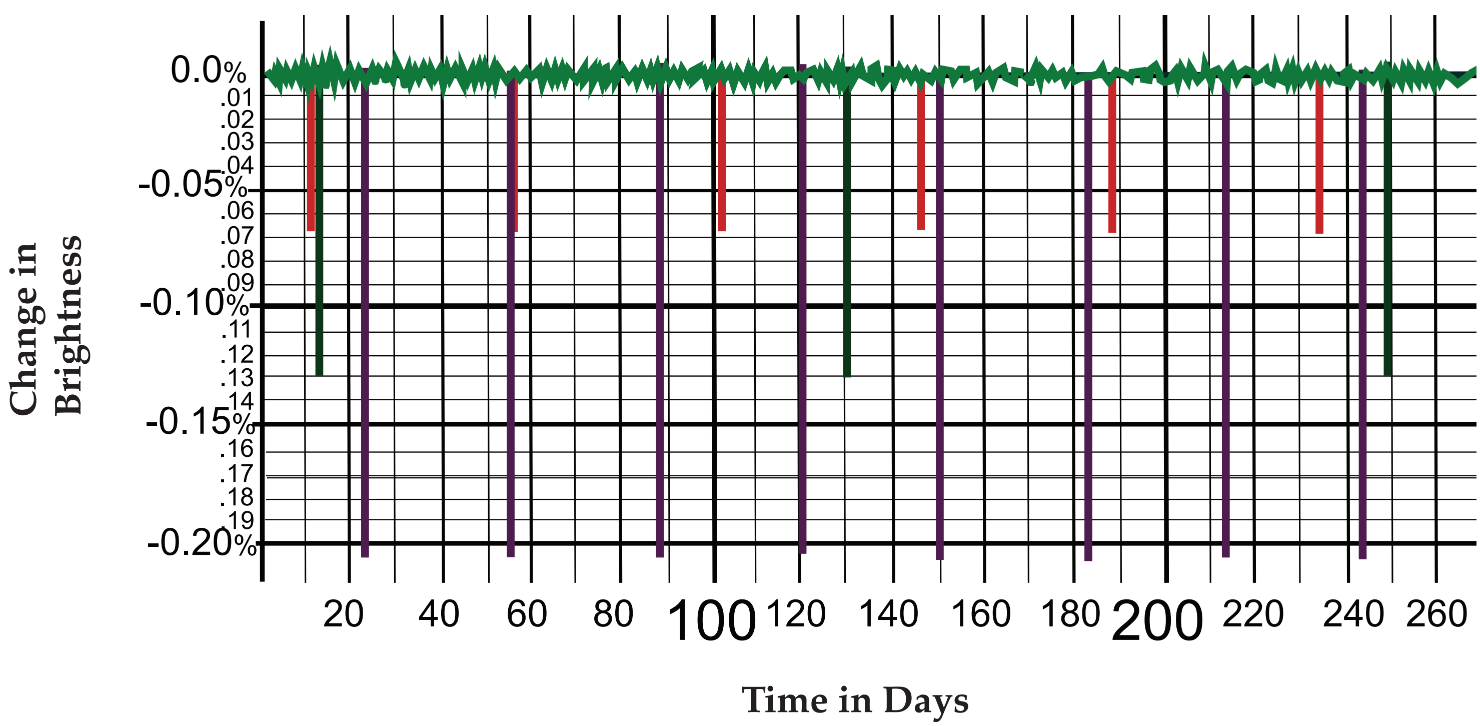


Transit Light Curves

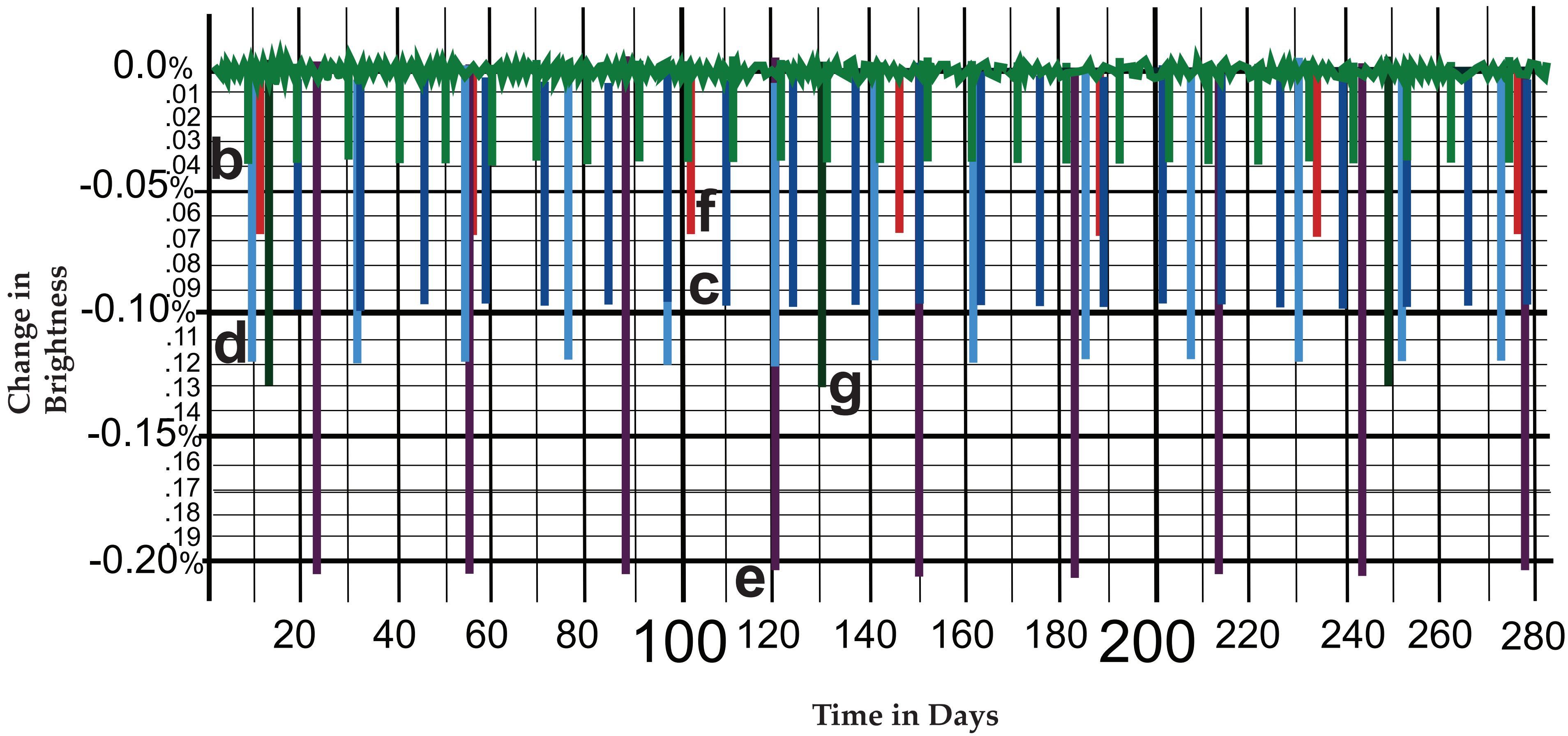
Kepler-11b, 11c, 11d



Kepler-11e, 11f, 11g



Kepler-11b, 11c, 11d, 11e, 11f, 11g



Analyzing Light Curves

Names: ____

____

Instructions: The "Transit Light Curves" are graphs of NASA's Kepler Mission's observations of stars. They show how the light level changes when a planet transits in front of a star. Study the light curves to find the period of the planet.The period is the time between transits and is year-length for a planet. Use "Kepler's 3rd Law Graphs" to find the "Orbital Distance" of the planet from its parent star.

The "Planet's Size" is found by measuring the "Change in Brightness," the small percentage drop in the light level as the planet transits. Calculate the planet's radius using the formula in the table below.

Orbital Distance

(from Kepler's 3rd Law graph)

Planet Name	**Period** Units ________	**Orbital Distance** Units ___________

Planet's Size

(planet radius using formula)

Planet	**Brightness** Drop of Z (%)	$\sqrt{Z}$	**Radius = 10 x** $\sqrt{Z}$ (in Earth radii)

Questions:

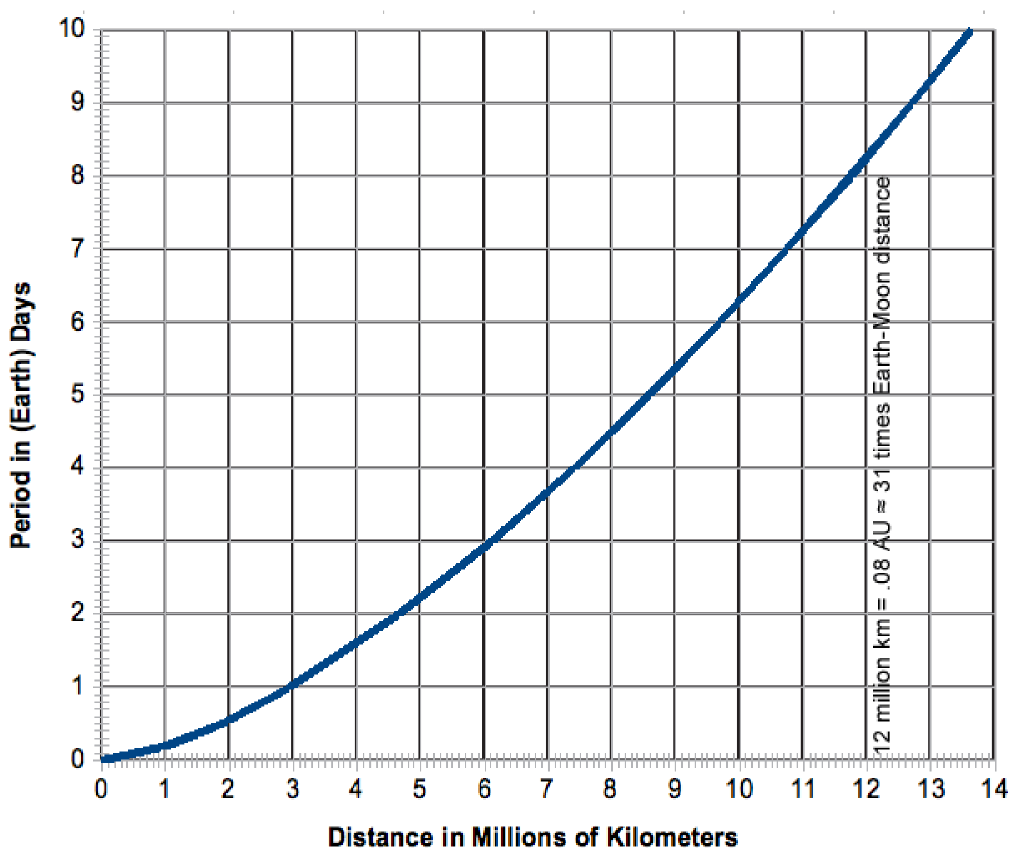
1. Which planet(s) are similar in size to Earth?

2. Jupiter's radius is about 11 times Earth's radius. Which planets are similar in size to Jupiter?

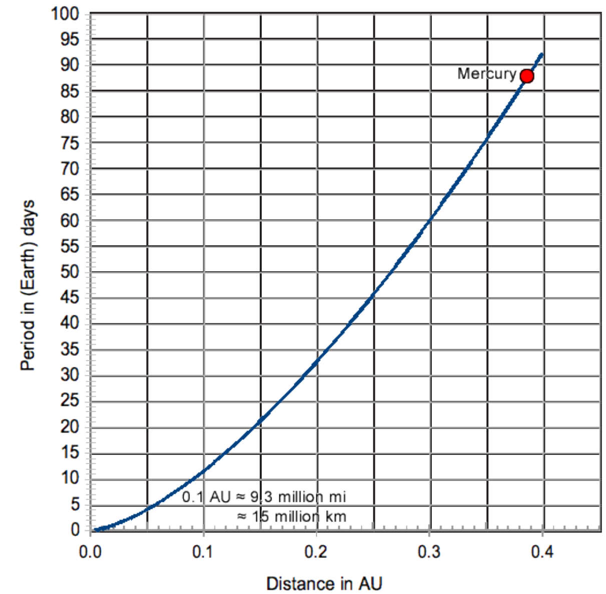
3. Describe the relationship between the period of the planets and their orbital distances.

Kepler's 3rd Law Graphs

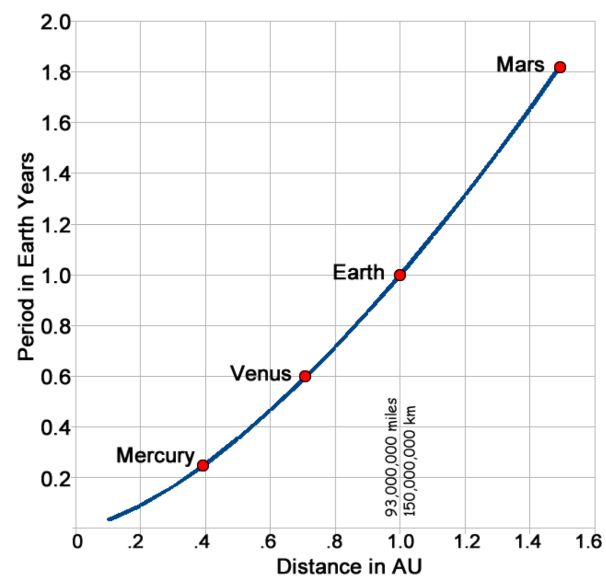
Kepler's 3rd Law Graph for Periods less than 10 days



Kepler's 3rd Law Graph for Periods Less Than 100 Days



Kepler's 3rd Law Graph for the Inner Solar System (periods less than 2 years)



Analyzing Light Curves: Calculated with Kepler's 3rd Law

Names: ______________________________

Instructions: The "Transit Light Curves" are graphs of NASA's Kepler Mission's observations of stars. They show how the light level changes when a planet transits in front of a star. Study the light curves to find the period of the planet. The period is the time between transits and equals year-length for a planet (T).

Like Johannes Kepler did, we express the planet's distance (R) in **A**stronomical **U**nits (AU). 1 AU is the average distance from the Earth to the Sun.

For stars the same size as the Sun, Kepler's 3rd Law is simply:

$$R^3 = T^2 \ * \quad \text{or} \quad R = \sqrt[3]{T^2 \cdot M_s}$$

For the Sun, Ms = 1. For other stars, Ms is mass in relationship to the Sun's mass.

The "Planet's Size" is determined by measuring the "Change in Brightness," the percentage drop in the light level as the planet transits. Calculate the planet's radius using the formula in the table below.

* *Note: There is actually a constant K implied in this equation that sets the units straight:*

$R^3/T^2 = K$ *where* $K = 1\ AU^3/Year^2$

Orbital Distance

from Kepler's 3rd Law

Planet	Period (T) Units: _______	T^2	Ms	$R = \sqrt[3]{T^2 M_s}$ Units: _______

Planet's Size

(radius using formula)

Planet	Brightness Drop of Z (%)	$\sqrt{Z}$	Radius = $10 \times \sqrt{Z}$ (in Earth radii)

CUBE ROOTS

Number	Cube Root	Number	Cube Root	Number	Cube Root
0.0025	0.136	0.09	0.448	0.32	0.684
0.0050	0.171	0.1	0.464	0.34	0.698
0.0075	0.196	0.12	0.493	0.36	0.711
0.0010	0.100	0.14	0.519	0.38	0.724
0.0100	0.215	0.16	0.543	0.4	0.737
0.02	0.271	0.18	0.565	0.5	0.794
0.03	0.311	0.2	0.585	0.6	0.843
0.04	0.342	0.22	0.604	0.7	0.888
0.05	0.368	0.24	0.621	0.8	0.928
0.06	0.391	0.26	0.638	1	1.000
0.07	0.412	0.28	0.654		
0.08	0.431	0.3	0.669		

For cube-root calculator instructions:
http://kepler.nasa.gov/education/cuberoot/

Questions:

1. Which planet(s) are similar in size to Earth?

2. Jupiter's radius is about 11 times Earth's radius. Which planets are similar in size to Jupiter?

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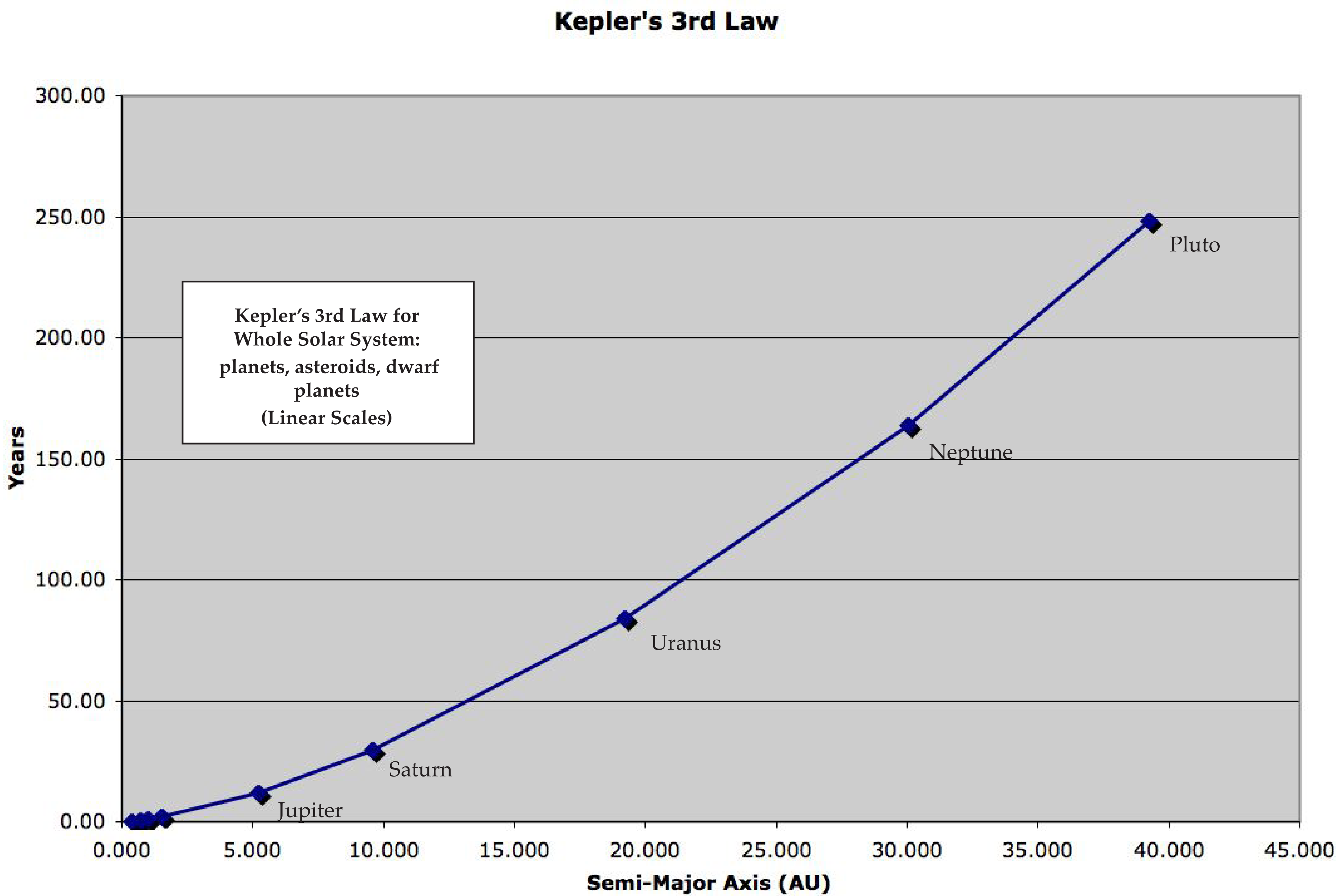
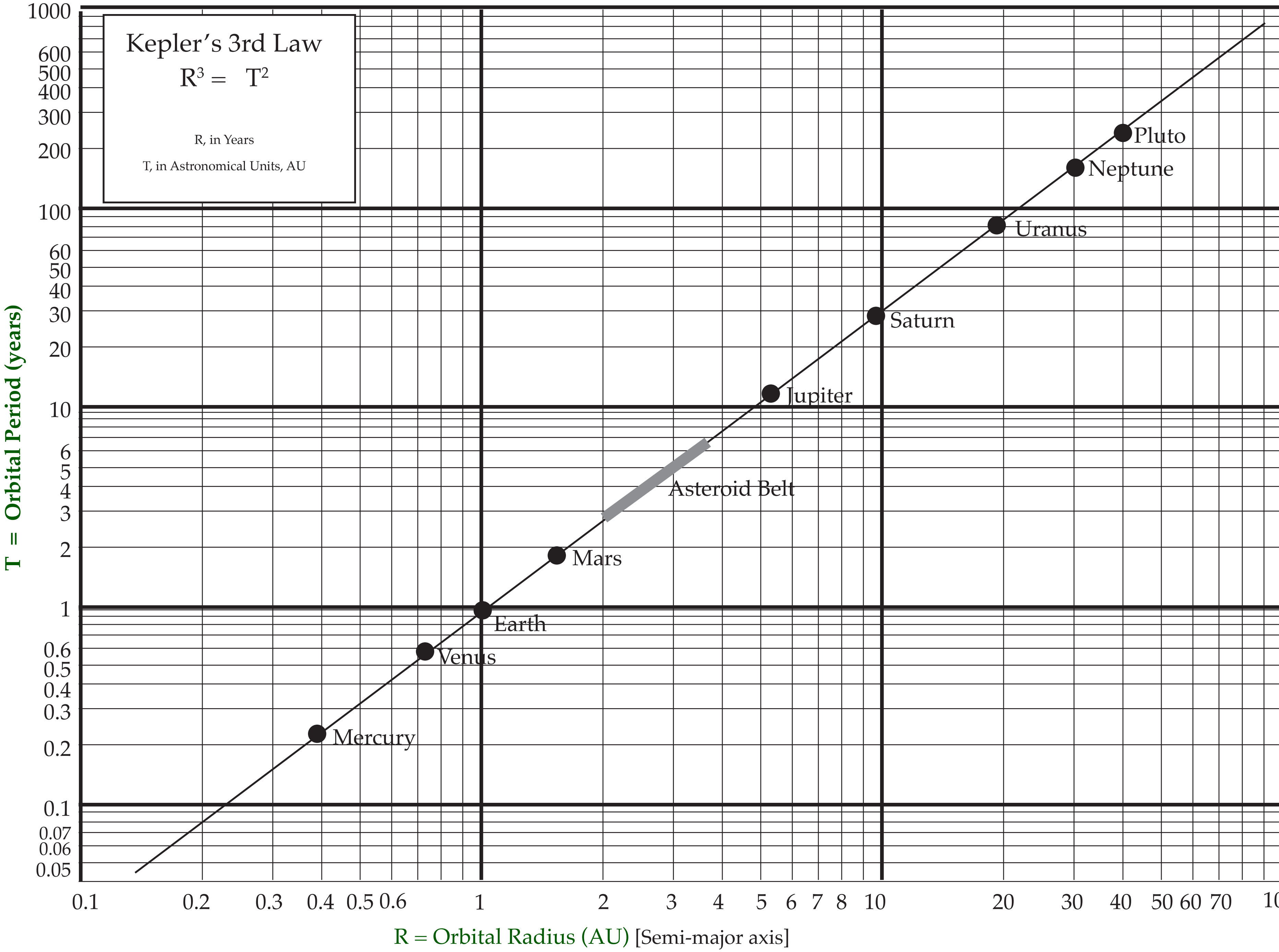

Kepler's 3rd Law
Kepler's 3rd Law for Whole Solar System: planets, asteroids, dwarf planets (Linear Scales)
Years
300.00
250.00
200.00
150.00
100.00
50.00
0.00
0.000
5.000
10.000
15.000
20.000
25.000
30.000
35.000
40.000
45.000
Semi-Major Axis (AU)
Pluto
Neptune
Uranus
Saturn
Jupiter

Keplers's 3rd Law Graph of Whole Solar System with Logarithmic Scales



Note: All objects -- planets, moons, asteroids, comets, meteoroids, dwarf planets -- all obey Kepler's 3rd Law.

Jeremiah Horrocks Makes First Observation of the Transit of Venus:

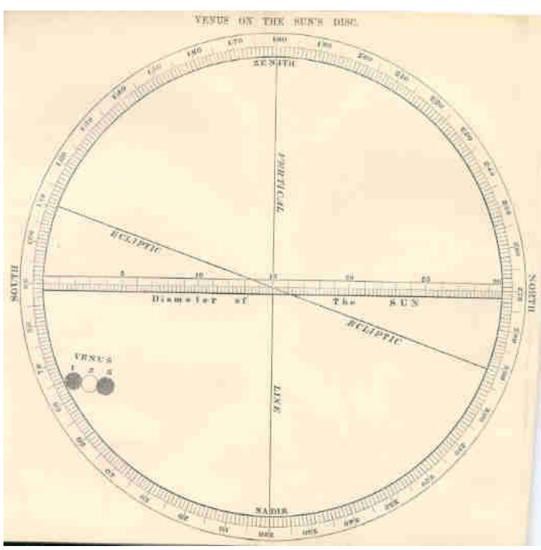
A transit of Venus across the Sun takes place when the planet Venus passes directly between the Sun and Earth, so that Venus blocks a small spot of the Sun's disk. Since the Sun is over 100 times larger in diameter than Venus, the spot is very small indeed.

An Englishman, Jeremiah Horrocks, made the first European observation of a transit of Venus from his home in Much Hoole, England, in the winter of 1639. Horrocks had read about Johannes Kepler who predicted transits in 1631 and 1761, and a near miss in 1639 when Venus would pass very close to the Sun, but not actually in front of it.

Horrocks made corrections to Kepler's calculation for the orbit of Venus and predicted that 1639 would not be a near miss, but an actual transit. He was uncertain of the exact time, but calculated that the transit would begin about 3:00 pm. He focused the image of the Sun through a simple telescope onto a card, where the image could be safely observed. After watching for most of the day with clouds obscuring the Sun often, he was lucky to see the transit as clouds cleared at about 3:15 pm, just half an hour before sunset. The observations allowed him to make a well-informed estimate as to the size of Venus, but more importantly, using geometry, to calculate the distance between the Earth and the Sun which had not been known accurately at that time. He was the first of many people who used transit observations to try to determine the distance from the Sun to the Earth.

More About Jeremiah Horrocks and His Observations:
Accounts of Horrocks' (Horrox's) observations were published in Latin, as was the common practice in the 17th century. "Venus in sole visa" was published many years after Horrocks' death (at age 22) by Johannes Hevelius. An English translation of Horrocks' notes on the transit are posted here:
http://www.venus-transit.de/1639/horrox.htm

A short account of his short life, and his observations of the transit of Venus:
http://www.transit-of-venus.org.uk/history.htm



Jeremiah Horrocks' drawing of the transit of Venus, 1639.

Jeremiah Horrocks observing the transit of Venus, artist's conception. (Public domain).

Transit Track Light Curves Key:
The sizes and distances that the students determine are unlikely to match exactly the "answers" provided below because any small difference between the percentage dip in the light or the length of the period for the planet will alter the outcome significantly.

Recommendation: Have students look up their planet, and find out the period and size from the Kepler Mission Discoveries table: ***http://kepler.nasa.gov/discoveries/*** Ask the students to explain how their answers are different, and more importantly, why? They should discuss accuracy of the data they had to work with, their ability to interpret the light curves from a paper copy, and the estimations built into the calculations (Earth radius is actually 109 times smaller the Sun, but we use 100 to derive the formula for converting the Z% to a radius.)

Data for Comparison for "Transit Tracks Light Curves"

Name	Period	Period Units	Distance	Distance Units	Radius in Earth radii
Kepler-1b	2.47	days	5,319,700	km	14.26
Kepler-2b	2.20	days	5,669,759	km	15.93
Kepler-3b	4.89	days	7,928,687	km	5.07
Kepler-4b	3.21	days	6,818,671	km	4.00
Kepler-5b	3.55	days	7,575,636	km	16.04
Kepler-6b	3.23	days	6,832,135	km	14.83
Kepler-7b	4.89	days	9,310,971	km	16.57
Kepler-8b	3.52	days	7,225,577	km	15.91
Kepler-9b	19.24	days	20,943,702	km	9.44
Kepler-9c	38.91	days	33,659,521	km	9.23
Kepler-10	.84	days	2,519,228	km	1.416
Mystery-b	1	Earth years	1	AU	1
Mystery-c	.66	Earth years	.76	AU	4
Kepler-11b	10.30	days	13,613,406	km	1.97
Kepler-11c	13.03	days	15,857,374	km	3.15
Kepler-11d	22.69	days	23,786,061	km	3.43
Kepler-11e	32.00	days	29,021,987	km	4.52
Kepler-11f	46.69	days	37,399,468	km	2.61
Kepler-11g	118.38	days	69,114,216	km	3.66

Data adapted (numbers are rounded) from data at:
Kepler Mission Discovery Table:
http://kepler.nasa.gov/discoveries/